Module 6, Solutions (16-18)

16) First, we factor $n^3 - n$ into three binomials:

$$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1) = (n - 1)n(n + 1)$$

Now we have $n^3 - n$ as a product of three consecutive numbers. Given three consecutive numbers, one of these numbers must be divisible by three. Also, if n-1 is not even, then n must be even – thus $(n - 1)n$ must be divisible by two. Since the number $n^3 - n$ is divisible by two and three it must also be divisible by six.

17) (*Proof by contradiction*) If such a square number exists, it is divisible by three (because whenever the sum of digits is divisible by three, then the number is divisible by three). However, if this number is a square, it then it must be divisible by 3x3 =9. But the number is not divisible by 9 (because if a number is divisible by nine, the sum of digits is divisible by nine).

18) $p^2 - 1 = (p - 1)(p + 1)$. (p-1), p and (p+1) are 3 consecutive numbers. As we saw in example 16) above, one of these numbers must be divisible by 3. Since p is a prime greater than 3, it is not divisible by 3. Thus either (p-1) or (p+1) must be divisible by 3, so $p^2 - 1 = (p - 1)(p + 1)$ must be divisible by 3.